

Downlink Capacity and Base Station Density in Cellular Networks

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Abstract

The cellular network is one of the most useful networks for wireless communications and now universally used. There have been a lot of analytic results about the performance of the mobile user at a specific location such as the cell center or edge, whereas very few results about the performance of the mobile user at an arbitrary location. In this paper, we use the stochastic geometry approach, and derive useful distributions and probabilities for cellular networks. Using these results, we analyze the downlink performance of the mobile user, e.g., outage probability at an arbitrary location considering the mobile user density. Under some assumptions, we can express these by closed form formulas. Our results will provide a framework for performance analysis of the wireless infrastructure with a high density of access points, which will significantly reduce the burden of network-level simulations.

Index Terms

Capacity, outage probability, stochastic geometry, downlink, wireless network.

I. INTRODUCTION

The capacity of the cellular networks has been a classical and important issue for efficient radio resource management [1]. The most improvement of the network capacity has come from reducing the cell size by installing more base stations such as femtocells [2], [3]. We may have a question, “How much does the network capacity increase as we install more base stations?” Unfortunately, answers to the question are not trivial, in particular when it comes to the case of

multiple interfering base stations and mobile users. So far, the only tractable approach is to rely on simulations, where various models on radio channels and the spatial distribution of base stations and users are used. In this paper, we tackle the issue to derive closed form formulas for quickly answering the questions.

Our key mathematical tool is the stochastic geometry, where base stations and mobile users can be modeled as independent homogeneous Poisson Point Processes (PPPs). The stochastic geometry has recently got much attention in particular for quantifying the co-channel interference in the wireless network (see [4] and literature therein). It has been applied to CDMA cellular networks [5], cellular networks with multi-cell cooperation [6], femtocells [7], cognitive radio networks [8] and CSMA/CA based wireless multihop networks [9].

In this paper, we derive the *downlink* capacity of a cellular network, as closed form formulas, and evaluate its correctness by means of simulations. The most relevant research to our work is the one by Andrews *et al.* [10]. In that paper, the authors used a PPP modeling for the base station distribution but did not consider the user density. Therefore, their results are useful for calculating the *area* outage probability, i.e., the probability that an arbitrary location is under outage due to the low signal-to-interference ratio. On the other hand, we derive the *user* outage probability, the one that an arbitrary user is under outage.

We assume base stations and mobile users are located with respective densities and radio channels fluctuate according to short-term fading and pathloss. The inter-cell interference is dependent on the frequency reuse factor but here we assume that every channel can be reused in every cell (i.e., the frequency reuse factor is 1). The rest of the paper contains how we derive our results (Propositions 1, 2, 3 and 4), which we hope to be useful in analyzing general wireless networks.

II. SYSTEM MODEL

Consider a downlink cellular network consisting of base stations (BSs) and mobile users (MUs). Almost all studies on cellular networks assumed that BSs are positioned regularly. However, in reality, it is not true and there are some random characteristics. To remedy the model, Andrews *et al.* [10] used PPP for the spatial distribution of the BSs.¹ We take the same

¹The authors insist that the PPP model for cellular networks has many advantages in comparison with the grid model. If the reader wants to get more information about the PPP model, please read [10] carefully.

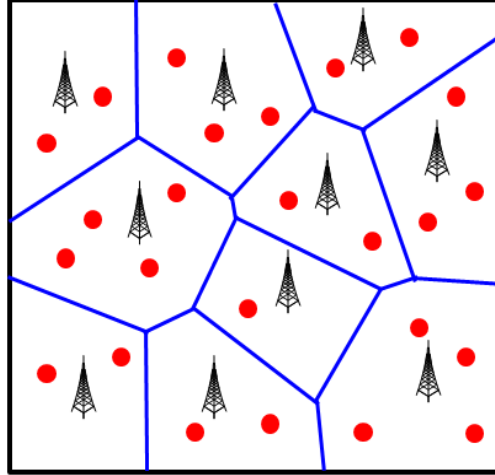


Fig. 1. The base stations and mobile users modeled as Poisson point process. The cell area of each base station forms a Voronoi tessellation.

approach for modeling the spatial distribution of the BSs. Besides, we consider the density of MUs, for which we also use PPP. One can argue that the MS distribution may not be best modeled as PPP. However, this is a tractable and reasonable approach as was also used in [11].

The spatial distribution of BSs follows PPP Φ_b with the density λ_b , over which MUs are positioned with PPP Φ_u with the density λ_u . Each MU is served by the nearest BS. This means that the cell area of each BS forms a *Voronoi tessellation* [12] as in Figure 1. We assume that the radio channel attenuation is dependent on pathloss and Rayleigh fading in our analysis (Section IV). Further, we consider log-normal shadowing as well in our simulation (Section V).

We consider only one resource block at a given time and assume that only one MU is scheduled in the resource block. In other words, if there are multiple MUs in the Voronoi cell of a BS, then the BS should serve only one of them in the resource block. The resource block can be interpreted as a time slot (in time division multiple access systems), a sub-carrier (in frequency division multiple access systems) or a scheduled slot (in code division multiple access systems). We assume that selection probabilities of the MUs within a Voronoi cell are equally likely (i.e., random selection with equal probability) for the fairness. On the other hand, there might be some BSs that do not have any MU to serve. In that case, the BSs will not transmit any signal (i.e., inactive).

III. INACTIVE BASE STATION PROBABILITY AND USER SELECTION PROBABILITY

In this section, we derive two important probabilities, *inactive BS probability* and *user selection probability*. The inactive BS probability refers to the probability that a randomly chosen BS does not have any MU in its Voronoi cell. This probability will be used for calculating the aggregate inter-cell interference in Section IV. The user selection probability denotes the one that a randomly chosen MU is assigned a resource block at the given time and is served by the nearest BS.

A. Inactive Base Station Probability

At a given time, there can be BSs that do not have any MU in their Voronoi cells. This happens when the BS density is high, e.g., femtocells. Those BSs are inactive. We start with the probability density function of the size of a Voronoi cell, which was derived by the Monte Carlo method [13]:

$$f_X(x) = \frac{3.5^{3.5}}{\Gamma(3.5)} x^{2.5} e^{-3.5x}, \quad (1)$$

where X is a random variable that denotes the size of a Voronoi cell normalized by the value $1/\lambda_b$. Using (1), we can derive the probability mass function of the number of MUs in an arbitrary Voronoi cell:

Lemma 1: *Let the random variable N denote the number of MUs in the Voronoi cell of a randomly chosen BS. Then, the probability mass function of N is*

$$P[N = n] = \frac{3.5^{3.5} \Gamma(n + 3.5) (\lambda_u/\lambda_b)^n}{\Gamma(3.5) n! (\lambda_u/\lambda_b + 3.5)^{n+3.5}}.$$

Proof: Using the law of total probability and the function (1), the probability mass function of N is given as

$$\begin{aligned} P[N = n] &= \int_0^\infty P[N = n|X = x] \cdot f_X(x) dx = \int_0^\infty \frac{\left(\lambda_u \frac{x}{\lambda_b}\right)^n}{n!} e^{-\lambda_u \frac{x}{\lambda_b}} \cdot f_X(x) dx \\ &= \frac{3.5^{3.5}}{\Gamma(3.5)} \frac{(\lambda_u/\lambda_b)^n}{n!} \int_0^\infty x^{n+2.5} e^{-(\lambda_u/\lambda_b + 3.5)x} dx = \frac{3.5^{3.5}}{\Gamma(3.5)} \frac{(\lambda_u/\lambda_b)^n}{n!} L_{x^{n+2.5}}(\lambda_u/\lambda_b + 3.5) \\ &= \frac{3.5^{3.5} \Gamma(n + 3.5) (\lambda_u/\lambda_b)^n}{\Gamma(3.5) n! (\lambda_u/\lambda_b + 3.5)^{n+3.5}}, \end{aligned}$$

where $L_{f(x)}(s)$ denotes the Laplace transform of $f(x)$. ■

Using Lemma 1, we derive the inactive BS probability as follows:

Proposition 1: *The probability ($p_{inactive}$) that a randomly chosen BS does not have any MU in its Voronoi cell is*

$$p_{inactive} = P[N = 0] = (1 + 3.5^{-1} \lambda_u / \lambda_b)^{-3.5}$$

B. User Selection Probability

Now we calculate the probability that a randomly chosen MU in a BS is selected for service at a given time. To derive the probability, we need the following property:

Lemma 2: *The probability density function ($f_Y(y)$) of the size of the Voronoi cell to which a randomly chosen MU belongs is*

$$f_Y(y) = \frac{3.5^{4.5}}{\Gamma(4.5)} y^{3.5} e^{-3.5y},$$

where Y is a random variable that denotes the size of the Voronoi cell normalized by the value $1/\lambda_b$.

Proof: Consider an arbitrary Voronoi cell and let $I \in \{0, 1\}$ denote the random variable that a randomly chosen MU is located in the Voronoi cell. If the randomly chosen MU is located in the Voronoi cell, then $I = 1$. Otherwise, $I = 0$. Consider the probability $P[I = 1 | X = x]$, where X is a random variable that denotes the size of an arbitrary Voronoi cell as in Equation (1). Using the fact that the probability is proportional to x , we can get the following equations:

$$P[I = 1 | X = x] = \frac{f_{X,I}(x, 1)}{f_X(x)} = cx \rightarrow f_{X,I}(x, 1) = cx f_X(x), \quad (2)$$

where c is a constant value. Note that $f_Y(y) = f_{X|I=1}(y)$ by definition. Therefore, we can derive $f_Y(y)$ as follows:

$$f_Y(y) = f_{X|I=1}(y) = \frac{f_{X,I}(y, 1)}{P[I = 1]} = \frac{cy f_X(y)}{P[I = 1]} = c' y f_X(y),$$

where c' is another constant value. Finally, using the fact that $\int_0^\infty f_Y(y) dy = 1$, we get the probability density function in this lemma. ■

Using Lemma 2, we derive the user selection probability as follows:

Proposition 2: *The probability ($p_{\text{selection}}$) that a randomly chosen MU is assigned a resource block at a given time and is served by the nearest BS is*

$$p_{\text{selection}} = \frac{1}{\lambda_u/\lambda_b} \left(1 - \left(1 + 3.5^{-1} \lambda_u/\lambda_b \right)^{-3.5} \right).$$

Proof: The user selection probability given the number of the other MUs (i.e., $N' = n$) is equal to $1/(n+1)$, and the location of the other MUs follows the reduced Palm distribution with the PPP Φ_u (Slivnyak's theorem [14]). Therefore, using the law of total probability, $p_{\text{selection}}$ is given as

$$\begin{aligned} p_{\text{selection}} &= \sum_{n=0}^{\infty} \frac{1}{n+1} \cdot P[N' = n] = \sum_{n=0}^{\infty} \frac{1}{n+1} \cdot \int_0^{\infty} P[N' = n | Y = y] \cdot f_Y(y) dy \\ &= \int_0^{\infty} \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{\left(\lambda_u \frac{y}{\lambda_b} \right)^n}{n!} e^{-\lambda_u \frac{y}{\lambda_b}} \cdot f_Y(y) dy = \int_0^{\infty} \frac{\lambda_b}{\lambda_u} y^{-1} \sum_{k=1}^{\infty} \frac{\left(\lambda_u \frac{y}{\lambda_b} \right)^k}{k!} e^{-\lambda_u \frac{y}{\lambda_b}} \cdot f_Y(y) dy \\ &= \int_0^{\infty} \frac{\lambda_b}{\lambda_u} y^{-1} \left(1 - e^{-\lambda_u \frac{y}{\lambda_b}} \right) \cdot f_Y(y) dy = \frac{3.5^{4.5}}{\Gamma(4.5)} \frac{\lambda_b}{\lambda_u} \int_0^{\infty} y^{2.5} e^{-3.5y} - y^{2.5} e^{-\left(3.5 + \frac{\lambda_u}{\lambda_b}\right)y} dy \\ &= \frac{3.5^{4.5}}{\Gamma(4.5)} \frac{\lambda_b}{\lambda_u} \left(L_{y^{2.5}}(3.5) - L_{y^{2.5}}\left(3.5 + \frac{\lambda_u}{\lambda_b}\right) \right) = \frac{1}{\lambda_u/\lambda_b} \left(1 - \left(1 + (3.5)^{-1} \lambda_u/\lambda_b \right)^{-3.5} \right). \end{aligned}$$

■

To verify our analysis, we conduct simulations with 10^5 independent samples of the location of BSs and MUs. We set the user density to $\lambda_u = 30$. We numerically calculate the probability density functions of the Voronoi cell size $f_Y(y)$ (Lemma 2), the inactive BS probability p_{inactive} (Proposition 1) and the user selection probability $p_{\text{selection}}$ (Proposition 2) in terms of the BS density λ_b . Figure 2 shows the results, which exactly coincide with Equation (1), Lemma 2, and Propositions 1 and 2.

IV. PERFORMANCE ANALYSIS OF CELLULAR NETWORKS

In this section, we analyze the capacity of cellular networks as a function of MU and BS density, and the target service quality. We define *service success probability* and *service capacity* as performance metrics. The service success probability refers to the probability that the cellular network succeeds in serving an arbitrary MU. It is composed with two parts, user selection

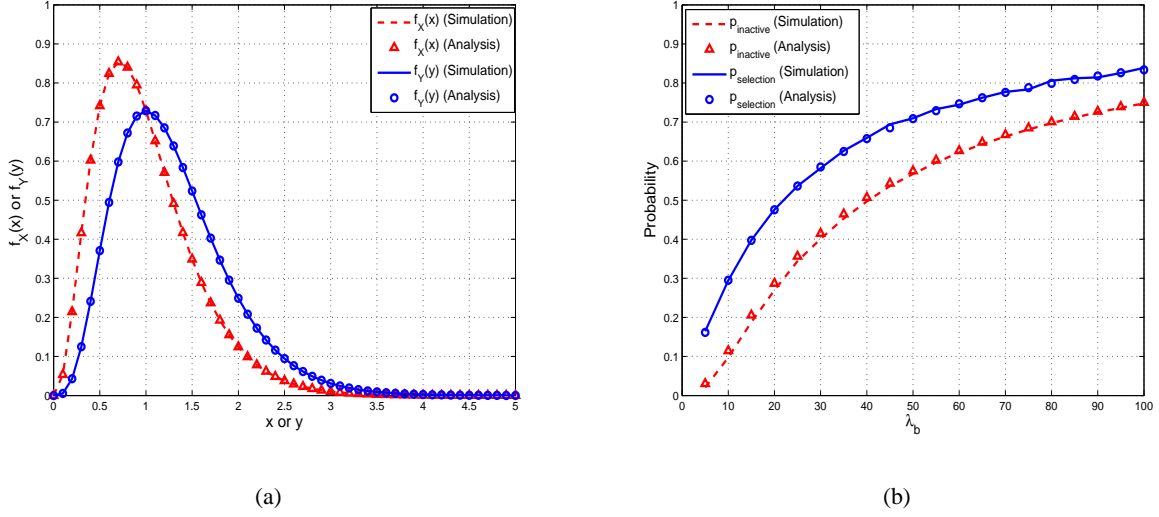


Fig. 2. (a) The probability density function $f_X(x)$ of the size of a Voronoi cell and the probability density function $f_Y(y)$ of the size of the Voronoi cell where a randomly chosen mobile user is located. (b) The inactive base station probability $p_{inactive}$ and the user selection probability $p_{selection}$ as a function of the base station density λ_b (the mobile users density is $\lambda_u = 30$).

probability (Proposition 2) and the transmission success probability which is defined in this section. The service capacity refers the MU density of success transmissions in the cellular network.

A. Service Success Probability

Service success probability ($p_{service}$) is defined as

$$p_{service} \triangleq p_{selection} \cdot p_{success}, \quad (3)$$

which means the probability that the cellular network succeeds in serving an arbitrary MU with some target signal-to-interference-noise ratio ($\hat{\gamma}$).² The transmission success probability ($p_{success}$) is the one that the MU's received signal to interference-noise ratio (γ) is higher than $\hat{\gamma}$. We derive the transmission success probability in the following lemma:

²The definition of $p_{service}$ is based on the assumption that $p_{selection}$ and $p_{success}$ are independent. Unfortunately, there is dependency between the two. If a MU is selected, then it is more likely to belong to a small cell, and thus interferes are likely to be closer. However, this dependency is negligible, which will be verified by the good match between theoretical and simulation results (Figure 3).

Lemma 3: *The transmission success probability ($p_{success}$) is*

$$p_{success} = \pi \lambda_b \int_0^\infty e^{-\pi \lambda_b \left(1 + \left(1 - (1 + 3.5^{-1} \lambda_u / \lambda_b)^{-3.5}\right) k\right) x - \frac{\hat{\gamma} \sigma_N^2 x^{\alpha/2}}{s}} dx,$$

where σ_N^2 and s denote the noise and the transmitted signal powers, respectively. The value α denotes the pathloss exponent and $k = \hat{\gamma}^{2/\alpha} \int_{\hat{\gamma}^{-2/\alpha}}^\infty 1/(1 + u^{\alpha/2}) du$.

Proof: From the result of [10], we get the transmission success probability as follows:

$$p_{success} = \pi \lambda_b \int_0^\infty e^{-\pi (\lambda_b + \lambda_i k) x - \frac{\hat{\gamma} \sigma_N^2 x^{\alpha/2}}{s}} dx, \quad (4)$$

where λ_i denotes the density of the BSs interfering with the MU. From Proposition 1, λ_i is equal to $\lambda_i = \lambda_b \cdot (1 - p_{inactive})$. Then, we get the result of this lemma. ■

The closed form formula ($p_{success}$) can be obtained when the pathloss exponent α is 4. Unfortunately, to the best of our knowledge, the other values of α do not give us such closed form. Using Proposition 2 and Lemma 3, we derive $p_{service}$ in the following proposition:

Proposition 3: *The service success probability ($p_{service}$) is*

$$p_{service} = \frac{\pi \lambda_b^2}{\lambda_u} \left(1 - (1 + 3.5^{-1} \lambda_u / \lambda_b)^{-3.5}\right) \cdot \int_0^\infty e^{-\pi \lambda_b \left(1 + \left(1 - (1 + 3.5^{-1} \lambda_u / \lambda_b)^{-3.5}\right) k\right) x - \frac{\hat{\gamma} \sigma_N^2 x^{\alpha/2}}{s}} dx.$$

If we assume that the noise is negligible (i.e., interference limited system) and $\alpha = 4$, then $p_{service}$ is reduced to the following closed form formula:

$$p_{service} = \frac{1 - (1 + 3.5^{-1} \lambda_u / \lambda_b)^{-3.5}}{\lambda_u / \lambda_b \left(1 + \left(1 - (1 + 3.5^{-1} \lambda_u / \lambda_b)^{-3.5}\right) k'\right)}, \quad (5)$$

where $k' = \sqrt{\hat{\gamma}} (\pi/2 - \arctan(1/\sqrt{\hat{\gamma}}))$.

B. Service Capacity

Service capacity ($C_{service}$) is defined as

$$C_{service} \triangleq \lambda_u \cdot p_{service}. \quad (6)$$

It is interpreted as the density of MUs with success transmissions. Using Proposition 3, we derive $C_{service}$ in the following proposition:

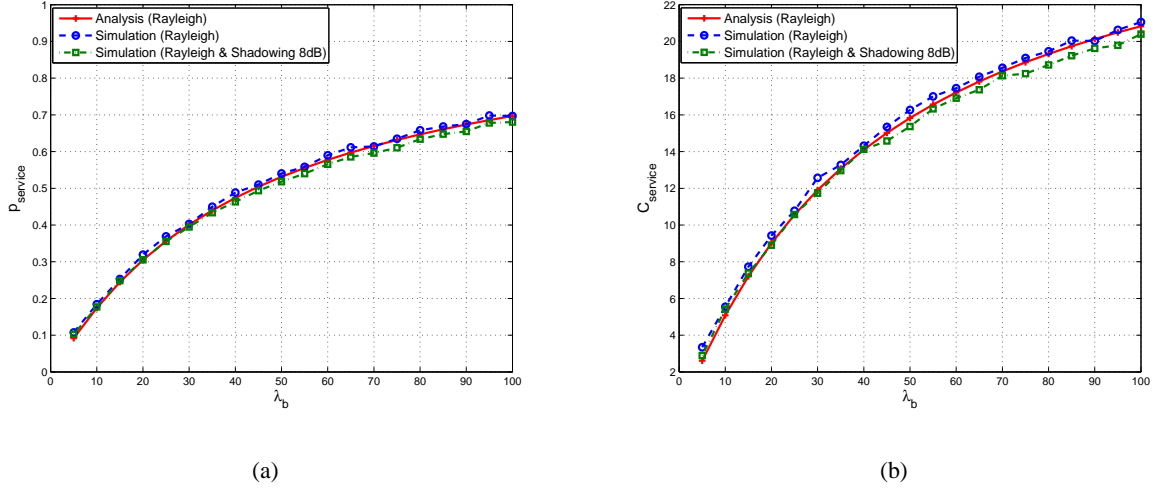


Fig. 3. Performance metrics as a function of the base station density λ_b : (a) The service success probability $p_{service}$. (b) The service capacity $C_{service}$ (the mobile user density is $\lambda_u = 30$, the pathloss exponent is $\alpha = 4$, the target signal to interference-noise ratio is $\hat{\gamma} = 0\text{dB}$, and interference limited system).

Proposition 4: *The service capacity ($C_{service}$) is*

$$C_{service} = \pi \lambda_b^2 \left(1 - \left(1 + 3.5^{-1} \lambda_u / \lambda_b \right)^{-3.5} \right) \cdot \int_0^\infty e^{-\pi \lambda_b \left(1 + \left(1 + 3.5^{-1} \lambda_u / \lambda_b \right)^{-3.5} \right) k} x - \frac{\hat{\gamma} \sigma_N^2 x^{\alpha/2}}{s} dx.$$

Again, if we assume that the noise is negligible and $\alpha = 4$, then $C_{service}$ is reduced to the following closed form formula:

$$C_{service} = \frac{\lambda_b \left(1 - \left(1 + 3.5^{-1} \lambda_u / \lambda_b \right)^{-3.5} \right)}{1 + \left(1 - \left(1 + 3.5^{-1} \lambda_u / \lambda_b \right)^{-3.5} \right) k'}, \quad (7)$$

where k' is given in (5).

To verify Propositions 3 and 4, we conduct simulations with 10^5 independent samples of the location of BSs and MUs. We assume an interference limited system and set the user density to $\lambda_u = 30$, the pathloss exponent $\alpha = 4$, the target signal to interference-noise ratio $\hat{\gamma} = 0\text{dB}$. We numerically calculate the service probability $p_{service}$ (Proposition 3) and the service capacity $C_{service}$ (Proposition 4). Figure 3 shows the results. In Proposition 3 and 4, we consider pathloss and Rayleigh fading in our channel model. On the other hand, we add the shadow fading in our simulation. Therefore, there is small gap between our analysis and simulation as the BS density increases. However, the general shape of the curves exactly match each other.

In Figure 3-(b), we see that the service capacity is a concave function of the number of BSs. This means the average quality of service may not increase rapidly with the installation of

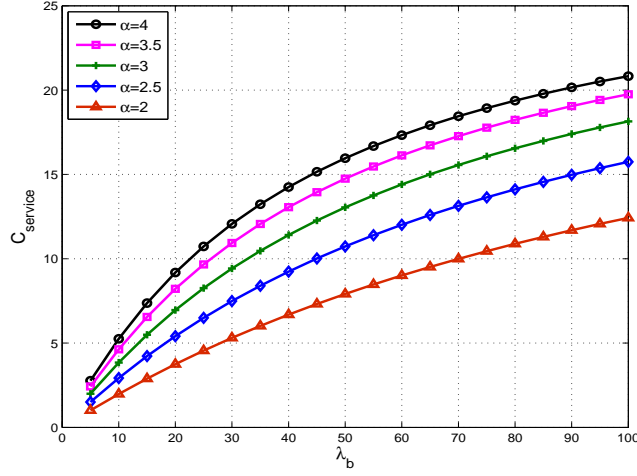


Fig. 4. The service capacity $C_{service}$ for various pathloss exponents α (the mobile user density is $\lambda_u = 30$, the target signal to interference-noise ratio is $\hat{\gamma} = 0\text{dB}$, and interference limited system).

additional BSs, after some point. As the number of BSs increases, the user selection probability will also increase due to the small number of MUs per cell. However, the increase the number of BSs will generate co-channel interference among the cells. Therefore, these two factors affect each other, which leads to decrease of the marginal capacity.

The numerical results (Figures 3) are based on the pathloss exponent $\alpha = 4$, where the closed form formula is available. For the other cases, we need to calculate the numerical integration part of Propositions 3 and 4. However, this burden is much less than the system level simulations. Figure 4 contains our results where the pathloss component varies between 2 and 4. In the figure, we see that the capacity of networks increases as the pathloss exponent becomes higher. This is due to the fact that the higher pathloss will filter co-channel interference among the cells [15]. On the other hand, we see that the behavior of diminishing the marginal capacity remains the same as in Figure 3-(b).

C. Asymptotic Cases

To get simpler closed form formulas, we consider two asymptotic cases. The first is the one that the density of BSs is much higher than that of MUs (i.e., $\lambda_b \gg \lambda_u$) like femtocells. In this case, the user selection probability can be approximated to one (i.e., $p_{selection} \approx 1$) and

the density of the transmitting BSs can be approximated to that of the MUs (i.e., $\lambda_i \approx \lambda_u$). Therefore, service success probability and service capacity is given as follows:

$$p_{service} \approx \pi \lambda_b \int_0^\infty e^{-\pi(\lambda_b + \lambda_u k)x - \frac{\gamma \sigma_N^2 x^{\alpha/2}}{s}} dx, \quad C_{service} \approx \pi \lambda_b \lambda_u \int_0^\infty e^{-\pi(\lambda_b + \lambda_u k)x - \frac{\gamma \sigma_N^2 x^{\alpha/2}}{s}} dx. \quad (8)$$

Moreover, if we assume that the noise is negligible and $\alpha = 4$, those are reduced to:

$$p_{service} \approx \frac{\lambda_b}{\lambda_b + \lambda_u k'}, \quad C_{service} \approx \frac{\lambda_b \lambda_u}{\lambda_b + \lambda_u k'}. \quad (9)$$

The second is the case that the density of the MUs is much higher than that of the BSs (i.e., $\lambda_u \gg \lambda_b$). This scenario is for the highly congested area like downtowns. In the case, inactive probability can be approximated to zero (i.e., $p_{inactive} \approx 0$) and the density of the transmitting BSs can be approximated to that of the existing BSs (i.e., $\lambda_i \approx \lambda_b$). Therefore, service success probability and service capacity is given as follows:

$$p_{service} \approx \frac{\pi \lambda_b^2}{\lambda_u} \int_0^\infty e^{-\pi \lambda_b (1+k)x - \frac{\gamma \sigma_N^2 x^{\alpha/2}}{s}} dx, \quad C_{service} \approx \pi \lambda_b^2 \int_0^\infty e^{-\pi \lambda_b (1+k)x - \frac{\gamma \sigma_N^2 x^{\alpha/2}}{s}} dx. \quad (10)$$

Similary, if we assume that the noise is negligible and $\alpha = 4$, those are reduced to:

$$p_{service} \approx \frac{\lambda_b}{\lambda_u (1 + k')}, \quad C_{service} \approx \frac{\lambda_b}{1 + k'}. \quad (11)$$

V. CONCLUSIONS

In this paper, we used the stochastic geometry approach and derived useful distributions and probabilities for cellular networks (Propositions 1, 2 and 3). Using these, we calculated the density of success transmissions in the downlink cellular network that was defined as the service capacity (Proposition 4).

Our analytic results will provide a fundamental framework for the performance analysis of cellular networks, which will significantly reduce the burden of network-level simulations. The limitation of our current work is as follows: First, we did not consider the shadow fading in the channel model of our analysis. Even though we verified our results using simulations, extension to the shadow fading case seems to be necessary. Second, the user selection is equally likely in each base station. However, we may consider more realistic scheduling algorithms into the analysis.

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